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1. Let  $u_1 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -1 \end{bmatrix}$ ,  $u_2 = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 1 \end{bmatrix}$ ,  $u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}$ ,  $u_4 = \begin{bmatrix} 5 \\ -3 \\ -1 \\ 1 \end{bmatrix}$  and  $x = \begin{bmatrix} 10 \\ -8 \\ 2 \\ 0 \end{bmatrix}$ .

- Show that  $\mathcal{B} = \{u_1, u_2, u_3, u_4\}$  form an orthogonal basis for  $R^4$ .
- Find an orthonormal basis for  $R^4$ . (Normalize the vectors in  $\mathcal{B}$ .)
- Write  $x$  as a linear combination of the basis elements in  $\mathcal{B}$ .
- Find  $[x]_{\mathcal{B}}$ .

2. Let  $u_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}$ ,  $u_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \\ 0 \end{bmatrix}$ ,  $v_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \\ -2 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} 7 \\ 1 \\ -2 \\ 6 \end{bmatrix}$ , and  $x = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ , and  $W = \text{Span}\{u_1, u_2\}$ .

- Compute  $\text{Proj}_W x$ .
- Show that the set  $\{v_1, v_2\}$  is another orthogonal basis for  $W$  and use it to compute  $\text{Proj}_W x$ .
- Find  $\text{dist}(x, W)$ .

3. Let  $v_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$ ,  $W = \text{Span}\{v_1, v_2\}$  and  $y = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$ .

- Find the orthogonal projection of  $y$  onto subspace  $W$ .
- What is the closest point to  $y$  in the subspace  $W$ ?
- Find the distance from  $y$  to  $W$ .
- Find an orthonormal basis for  $W$ .

4. Let

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix}, y = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

and  $W = \text{Span}\{u_1, u_2, u_3\}$ .

- Find the orthogonal projection of  $y$  onto subspace  $W$ .
- What is the closest point to  $y$  in the subspace  $W$ ?
- Find the distance from  $y$  to  $W$ .
- Find an orthonormal basis for  $W$ .